Sains Malaysiana 43(10)(2014): 1583–1590

MHD Mixed Convection Peristaltic Flow with Variable Viscosity and Thermal Conductivity

(Aliran Peristalsis Perolakan MHD Bercampur dengan Kelikatan yang Berubah-ubah dan Kekonduksian Terma)

T. HAYAT, F.M. ABBASI*, B. AHMAD & A. ALSAEDI

ABSTRACT

This article concerns with a mixed convection peristaltic flow of an electrically conducting fluid in an inclined asymmetric channel. Analysis has been carried out in the presence of Joule heating. The fluid viscosity and thermal conductivity are assumed to vary as a linear function of temperature. A nonlinear coupled governing system is computed. Numerical results were presented for the velocity, pressure gradient, temperature and streamlines. Heat transfer rate at the wall is computed and analyzed. Graphs reflecting the contributions of embedded parameters were discussed.

Keywords: Joule heating; mixed convection; variable thermal conductivity; variable viscosity

ABSTRAK

Artikel ini berkaitan dengan aliran peristalsis perolakan yang bercampur dengan cecair elektrik yang dijalankan di saluran asimetri cenderung. Analisis telah dijalankan dengan kehadiran pemanas Joule. Kelikatan cecair dan kekonduksian terma dianggap berubah sebagai fungsi linear suhu. Sistem pengelasan terganding tak linear dikira. Keputusan kajian berangka dibentangkan bagi halaju, kecerunan tekanan, suhu dan arus. Kadar pemindahan haba pada dinding dikira dan dianalisis. Graf yang mencerminkan sumbangan parameter yang tertanam dibincangkan.

Kata kunci: Kekonduksian terma yang berubah-ubah; kelikatan yang berubah-ubah; pemanas Joule; perolakan bercampur

INTRODUCTION

It is commonly accepted now that peristaltic flows are caused by the propagation of waves along the flexible boundaries of channel or tube. Such flows in physiology are represented by food movement in the digestive tract, urine transport from kidney to bladder, semen movement in vas deferens, movement of lymphatic fluids in lymph vessels, bile flow from the gall bladder into the duodenum, vasomotion of blood cells, movement of ovum in the female fallopian tube and transport of spermatozoa in the ductus efferents. Peristaltic motion in the industrial applications are employed in the transport of corrosive and noxious fluids, roller and finger pumps, hose pumps, tube pumps, dialysis machines and heart-lung machines. Latham (1966) and Shapiro et al. (1969) discussed the peristalsis of viscous fluids through theoretical and experimental approaches. Afterwards a wealth of literature related to peristaltic pumping of viscous and non-Newtonian fluids exists in view of one or more assumptions of long wavelength, low Reynolds number, small wave number and small amplitude ratio. Few recent investigations in this direction may be mentioned by the following references (Abd Elmaboud & Mekheimer Kh. 2011; Mekheimer Kh. et al. 2014, 2013a, 2013b; Hayat et al. 2014a; Hina et al. 2013; Pandey & Tripathi 2012; Tripathi et al. 2011) and many related attempts therein.

None of the above attempts deal with the influence of heat transfer in peristaltic transport of fluids. This concept has key roles in the analysis of tissues, hemodialysis and oxygenation. The recent progress in the application of heat (hyperthermia), radiation (laser therapy) and coldness (cryosurgery), as means to destroy undesirable tissues including cancer have stimulated much interest in mathematical modeling for properties of tissue. Moreover, radiofrequency therapy has significance in the treatment of diseases like tissue coagulation, the liver cancer, the lung cancer and reflux of stomach acid. Having such preference in mind, some authors (Abbasi et al. 2014a; Hayat et al. 2014b, 2014c, 2014d; Mekheimer Kh. et al. 2010; Srinivas et al. 2011) analyzed the peristaltic transport with heat transfer. In the aforementioned studies the authors have taken into account the constant fluid properties e.g. viscosity and thermal conductivity. However several applications in engineering occur at high temperature through variable thermal conductivity and fluid viscosity. Few such processes include nuclear power plants, pumps operated at high temperatures, in turbines, rockets, missile technologies and space vehicles. Even blood, basic living ingredient for animals is very sensitive to temperature changes and slight change in temperature may cause irreversible damage during dialysis or in heart lung machines. This ensures the knowledge of heat transfer with variable fluid viscosity and thermal conductivity. Although one can find few studies on the peristaltic transport with variable viscosity fluid (Abbasi et al. 2014b; Hayat & Abbasi 2011; Hayat et al. 2011) and but to the best of our knowledge no literature is yet available on the peristaltic transport of fluid possessing variable thermal conductivity. Here we intend to study peristaltic transport of viscous fluid with variable viscosity and thermal conductivity. An electrically conducting fluid is taken in an inclined channel. The Joule heating and mixed convection effects are included. Problem is modeled using long wavelength approximation. The governing non-linear coupled system is solved numerically. Obtained numerical data is analyzed through graphs. Numerical values of heat transfer rate at the wall for different embedded parameters are tabulated. Important points of this study are summarized at the end.

MATHEMATICAL ANALYSIS

We examine the peristaltic transport of viscous fluid in an inclined asymmetric channel of width $(d_1 + d_2)$. The channel is inclined at an angle α. The $\bar{X}$ – axis is chosen along the length of the channel and $\bar{Y}$ – axis is taken normal to the $\bar{X}$ – axis. A uniform magnetic field of strength B_0 acts parallel to the $\bar{Y}$ – axis. The effects of induced magnetic field are neglected under low magnetic Reynolds number approximation. Electric field effects are also negligible. The viscosity and thermal conductivity are temperature dependent. The following forms of waves propagate along the channel walls,

$$\bar{H}_1(\bar{X},\bar{t}) = d_1 + \xi_1,$$
$$\bar{H}_2(\bar{X},\bar{t}) = -d_2 - \xi_2$$
$$\xi_1 = a_1 \cos\left(\frac{2\pi}{\lambda}(\bar{X} - c\bar{t})\right),$$
$$\xi_2 = b_1 \cos\left(\frac{2\pi}{\lambda}(\bar{X} - c\bar{t}) + \phi\right),$$

in which $\bar{t}$ is time, $\bar{H}_1$ and $\bar{H}_2$ represent the upper and lower walls of channel, a_1 and b_1 are the amplitude of the waves at respective walls, ϕ the phase difference of two waves and c and λ are the speed and wavelength of the waves, respectively. Appropriate velocity field for this problem is $\bar{V} = \left[\bar{U}(\bar{X},\bar{Y},\bar{t}), \bar{V}(\bar{X},\bar{Y},\bar{t}), 0\right]$.

We consider $(\bar{u},\bar{v})$ and $\bar{p}$ as the velocity components and pressure in the wave frame $(\bar{x},\bar{y})$. The transformations between laboratory and wave frames are:

$$\bar{x} = \bar{X} - c\bar{t},\ \bar{y} = \bar{Y},\ \bar{u} = \bar{U} - c,\ v = \bar{V},\ \bar{p}(\bar{x},\bar{y}) = \bar{P}(\bar{X},\bar{Y},\bar{t}), \tag{1}$$

where $(\bar{U},\bar{V})$ and $\bar{P}$ are the velocity components and pressure in the laboratory frame. The conservation laws of mass and momentum in wave frame can be expressed as,

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{2}$$

$$\rho\left((\bar{u}+c)\frac{\partial}{\partial \bar{x}} + \bar{v}\frac{\partial}{\partial \bar{y}}\right)(\bar{u}+c) = -\frac{\partial \bar{p}}{\partial \bar{x}} + 2\frac{\partial}{\partial \bar{x}}\left(\bar{\mu}(T)\frac{\partial \bar{u}}{\partial \bar{x}}\right) + \frac{\partial}{\partial \bar{y}}\left[\bar{\mu}(T)\left(\frac{\partial \bar{v}}{\partial \bar{x}} + \frac{\partial \bar{u}}{\partial \bar{y}}\right)\right] - \sigma B_0^2(\bar{u}+c) + \rho g \alpha^* (T - T_m)\sin\alpha + \rho g \sin\alpha, \tag{3}$$

$$\rho\left((\bar{u}+c)\frac{\partial}{\partial \bar{x}} + \bar{v}\frac{\partial}{\partial \bar{y}}\right)(\bar{v}) = -\frac{\partial \bar{p}}{\partial \bar{y}} + 2\frac{\partial}{\partial \bar{y}}\left(\bar{\mu}(T)\frac{\partial \bar{v}}{\partial \bar{y}}\right) + \frac{\partial}{\partial \bar{x}}\left[\bar{\mu}(T)\left(\frac{\partial \bar{v}}{\partial \bar{x}} + \frac{\partial \bar{u}}{\partial \bar{y}}\right)\right] - \rho g \alpha^* (T - T_m)\cos\alpha + \rho g \cos\alpha, \tag{4}$$

in which ρ is the density of fluid, σ is the electric conductivity, $\bar{\mu}(T)$ is the temperature dependent dynamic viscosity, g is the acceleration due to gravity, T_m is the mean value of the temperature of both the channel walls and α^* is the thermal expansion coefficient.

Defining the dimensionless quantities,

$$x = \frac{\bar{x}}{\lambda},\ y = \frac{\bar{y}}{d_1},\ u = \frac{\bar{u}}{c},\ v = \frac{\bar{v}}{c\delta},\ \delta = \frac{d_1}{\lambda},$$
$$h_1 = \frac{\bar{H}_1}{d_1},\ h_2 = \frac{\bar{H}_2}{d_1},\ d = \frac{d_2}{d_1},\ p = \frac{d_1^2 \bar{p}}{c\lambda\mu_0},$$
$$\upsilon = \frac{\mu_0}{\rho},\ \mu(\theta) = \frac{\bar{\mu}(T)}{\mu_0},\ M^2 = \left(\frac{\sigma}{\mu_0}\right)^2 B_0^2 d_1^2,$$
$$t = \frac{c\bar{t}}{\lambda},\ Gr = \frac{\rho g \alpha^* (T_1 - T_0) d_1^2}{\mu_0 c},$$
$$\mathrm{Re} = \frac{\rho c d_1}{\mu_0},\ Fr = \frac{c^2}{g d_1},\ u = \psi_y,\ v = -\psi_x, \tag{5}$$

and adopting long wavelength and low Reynolds number approach we have in terms of stream function ψ the following equations,

$$p_x = \frac{\partial}{\partial y}\left[\mu(\theta)\frac{\partial^2 \psi}{\partial y^2}\right] - M^2\left(\frac{\partial \psi}{\partial y} + 1\right) + Gr\theta \mathrm{Sin}\alpha + \frac{\mathrm{Re}\,\mathrm{Sin}\alpha}{Fr}, \tag{6}$$

$$p_y = 0, \tag{7}$$

and the continuity equation is identically satisfied. Here, $h_{1,2}$ are the dimensionless wall shapes, M is the Hartman number, Gr is the Grashoff number, Re is the Reynolds number, Fr is the Froud number and ψ is the stream function. Equation (7) indicates that $p \neq p(y)$ and compatibility equation is given by

$$0 = \frac{\partial^2}{\partial y^2}\left[\mu(\theta)\frac{\partial^2 \psi}{\partial y^2}\right] - M^2\left(\frac{\partial^2 \psi}{\partial y^2}\right) + Gr\frac{\partial \theta}{\partial y}\sin\alpha. \tag{8}$$

Defining η and F as the dimensionless mean flows in laboratory and wave frames we have

$$\eta = F + 1 + d \tag{9}$$

where F is given by

$$F = \int_{h_2}^{h_1} \frac{\partial \psi}{\partial y} dy = \psi(h_1) - \psi(h_2). \tag{10}$$

Dimensionless boundary conditions in terms of stream function are,

$$\psi = \frac{F}{2},\ \psi_y = -1 \ \text{at} \ y = h_1,$$
$$\psi = -\frac{F}{2},\ \psi_y = -1 \ \text{at} \ y = h_2. \tag{11}$$

Heat Transfer Analysis

We assumed that the walls at h_1 and h_2 have temperatures T_0 and T_1 respectively. The energy equation in presence of viscous dissipation and Joule heating effects are

$$\rho C_p\left((\bar{u}+c)T_{\bar{x}} + \bar{v}T_y\right) = \bar{\nabla}.\left[\bar{K}(T)(\nabla T)\right]$$
$$+\bar{\mu}(T)\left[2\left\{\left(\frac{\partial \bar{u}}{\partial \bar{x}}\right)^2 + \left(\frac{\partial \bar{v}}{\partial \bar{y}}\right)\right\}^2 + \left(\frac{\partial \bar{v}}{\partial \bar{x}} + \frac{\partial \bar{u}}{\partial \bar{y}}\right)^2\right]$$
$$+\sigma B_0^2 (\bar{u}+c)^2, \tag{12}$$

in which T is the fluid temperature, C_p is the specific heat, K is the dimensional temperature dependent thermal conductivity and $\bar{\mu}(T)$ is the temperature dependent viscosity. The dimensionless energy equation in view of aforementioned approximations becomes

$$\frac{\partial}{\partial y}\left\{K(\theta)\theta_y\right\} + Br\mu(\theta)(\psi_{yy})^2 + BrM^2\left(\frac{\partial \psi}{\partial y} + 1\right)^2 = 0, \tag{13}$$

where Br is the Brinkman number, θ the dimensionless temperature, Pr the Prandtl number and E the Eckert number. These quantities are defined as follows:

$$\theta = \frac{T - T_m}{T_1 - T_0}, K(\theta) = \frac{\bar{K}(T)}{K_0}, Br = \Pr E,$$
$$Pr = \frac{\mu_0 C_p}{K_0}, E = \frac{c^2}{C_p(T_1 - T_0)}. \tag{14}$$

Dimensionless boundary conditions for the temperature are given by

$$\theta = -\frac{1}{2} \ \text{at} \ y = h_1,\ \theta = \frac{1}{2} \ \text{at} \ y = h_2. \tag{15}$$

Temperature dependent viscosity and thermal conductivity are taken in the form,

$$\mu(\theta) = 1 - \beta\theta, K(\theta) = 1 + \varepsilon\theta, \tag{16}$$

in which β and ε are the dimensionless viscosity and thermal conductivity coefficients respectively with (β, ε) << 1. Also note that μ_0 and K_0 are the viscosity and thermal conductivity of the fluid when the temperature of the fluid approaches the mean temperature. Also the cases of constant viscosity and thermal conductivity can be recovered by choosing β and ε equal to zero. The values of h_1 and h_2 are

$$h_1(x) = 1 + a\cos(2\pi x) \text{ and } h_2(x) = -d - b\cos(2\pi x + \phi).$$

We note that (13) is nonlinear in the present analysis. Hence in this study we have nonlinear coupled system. Analytic solution of the resulting nonlinear coupled system seems difficult. Hence the numerical solutions using shooting method in Mathematica is preferred here. The step sizes for x and y are chosen 0.01. The flow quantities of interest will be analyzed in the next section.

Graphical Analysis

This section aims to analyze the numerical results for influential parameters. Plots of pressure gradient, axial velocity, temperature and stream function are obtained and examined. Numerical values of the heat transfer rate are given in Table 1.

TABLE 1. Numerical values of heat transfer rate at the wall (h_1) when Re 3, $Fr = 2$, $Br = 0.2$, $x = 0$, $Gr = 2$, $Pr = 0.5$, $a = 0.4$, $b = 0.3$, $\phi = \pi/4$, $d = 0.7$ and $\eta = 1.2$.

β	ε	α	M	$-\theta'(h_1)$
0.0	0.1	$\pi/6$	0.5	1.0603
0.4				1.0257
0.6				1.0051
0.1	0.0			0.9893
	0.1			1.0524
	0.2			1.1258
	0.1	0		1.1278
		$\pi/4$		1.0247
		$\pi/2$		0.9892
		$\pi/6$	0.0	0.9857
			0.4	1.0284
			0.8	1.1567

It is obvious from Figure 1 that pressure gradient for constant viscosity fluid is less when compared with variable viscosity fluid. Temperature dependent thermal conductivity also increases the value of pressure gradient. Such results are obvious from Figure 1(a)-1(b). An increase in β and ε also increases the pressure gradient. Pressure gradient decreases by increasing Hartman and Froud numbers. However this change in pressure gradient due to M and Fr is sufficiently large when compared with β and ε (Figure 1(d) and 1(f)). As the value of α varied from 0 to π / 2 the pressure gradient tends to attain significantly large value which highlights the fact that pressure gradient is higher in a vertical channel when compared to horizontal channel (Figure 1(c)). For assisting flow (+ve value of acceleration due to gravity g) pressure gradient is larger than the resisting flow (Figure 1(e)).

Plots of axial velocity are given in the Figure 2(a)-2(d). These figures shows that velocity tends to follow parabolic path with maximum value occurring near the center of channel. Figure 1(a) shows that when the viscosity parameter β is taken zero i.e. the velocity for constant viscosity fluid is seen to have maximum value near the center of channel whereas it tends to shift towards the lower wall for non-zero values of the viscosity parameter. Moreover a slight increase in the maximum value of velocity is observed when β increases. Channel inclination angle α and β are seen to have almost similar effect on the velocity (Figure 2(b)). Maximum value of velocity in inclined channel is shifted towards the lower wall. Increase in the strength of applied magnetic field decreases

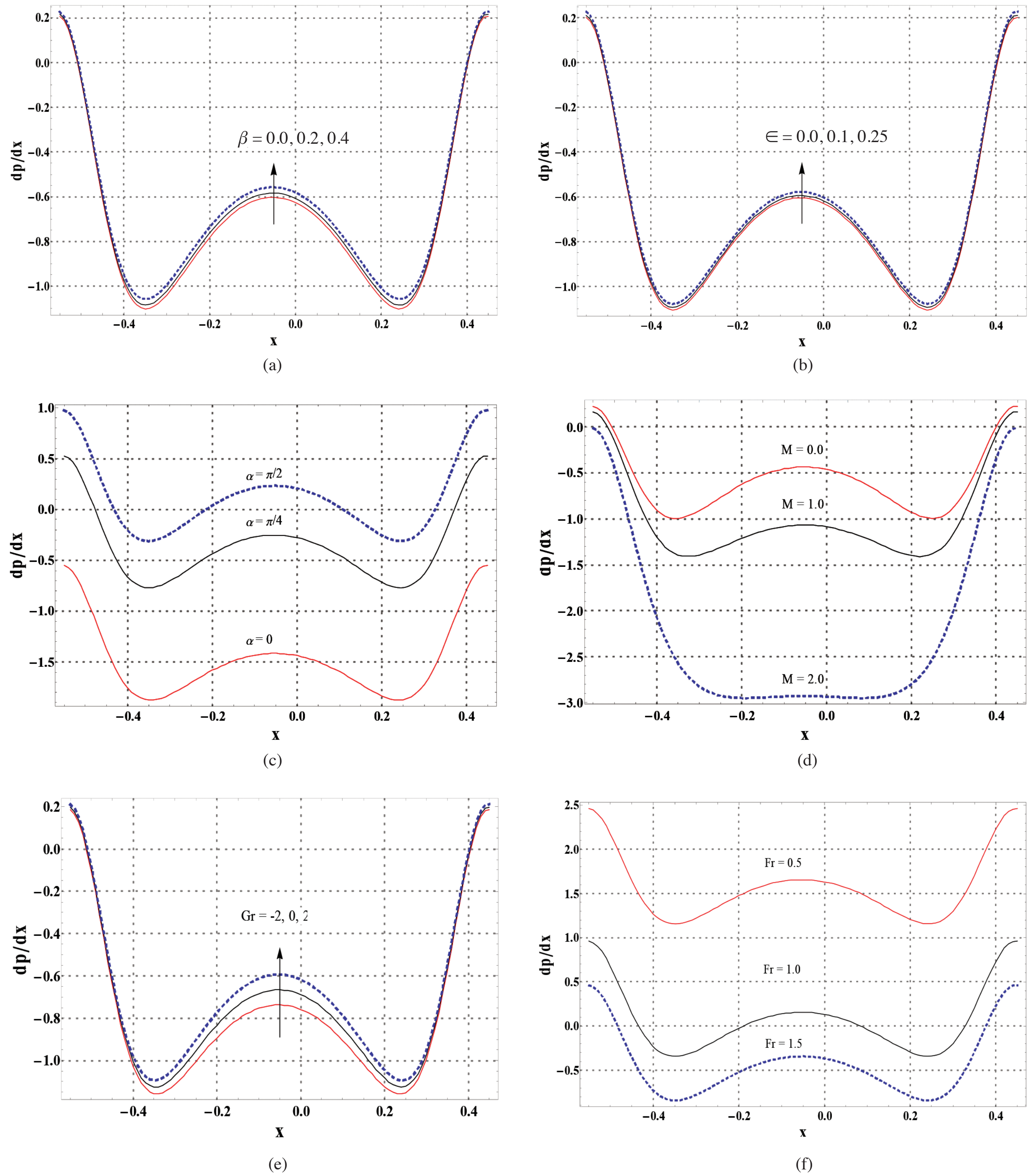

FIGURE 1(a)-1(f). Development of pressure gradient in the axial direction the velocity. This change in velocity is maximum when the Hartman number is taken between 1 and 2 (Figure 2(c)). Change in the value of Grashoff number varies the symmetry of the velocity profile relative to the channel i.e. velocity has maximum value near the center for resisting flow and this changes when Gr has +ve value. The effect of Grashoff number on the velocity also depends on the channel inclination which is well justified physically (Figure 2(d)). It is further noted that ε has no effect on the axial velocity.

Behavior of dimensionless temperature through β, ε, M, α, Gr and Br are examined in Figure 3(a)-3(f). Figure 3(a) depicts that an increase in the viscosity parameter decreases the temperature. This decay in temperature subject to decrease in β is uniform throughout the channel. Figure 3(b) analyzes the impact of variable thermal conductivity parameter ε on the temperature. The effect of ε on the temperature is noteworthy in the sense that it is not uniform throughout the channel. It means that near the lower wall (h_2) an increase in the value of ε decreases the value of temperature of the fluid whereas near the upper wall (h_1) it increases the value of θ. Since ε yields the perturbation in the thermal conductivity of fluid so increase in this parameter gives rise to increase in the ability of the fluid to dissipate or absorb heat. Hence when temperature of fluid is higher than the temperature of the boundary, an increase in ε results in reduction of the temperature of fluid and vice versa.

Increase in the Hartman number increases the value of θ. This fact is primarily due to consideration of Joule heating (Figure 3(c)). Channel inclination also has non-uniform effect on the temperature. Increase in α near the lower wall increases the temperature but such increase is not very significant. Similarly the Grashoff number also has very less and non-uniform effect on the temperature (Figure 3(d) and 3(e)). Figure 3(f) shows that an increase in Br increases the temperature θ significantly. Also when Br is increased, the temperature plots become non-linear. Streamlines for β and ε are plotted in Figures 4 and 5. It is seen that the size of trapped bolus decreases with an increase in β. However ε has no significant effect on the size of bolus.

Table 1 gives the numerical values of heat transfer rate at the wall $\theta'(h_1)$ for different values of the embedded parameters. The results showed that an increase in the value of viscosity parameter β reduces the heat transfer rate but such rate increases through an increase in ε.The transfer rate decreases via inclined nature of channel. Further increase in Hartman number increases the transfer rate.

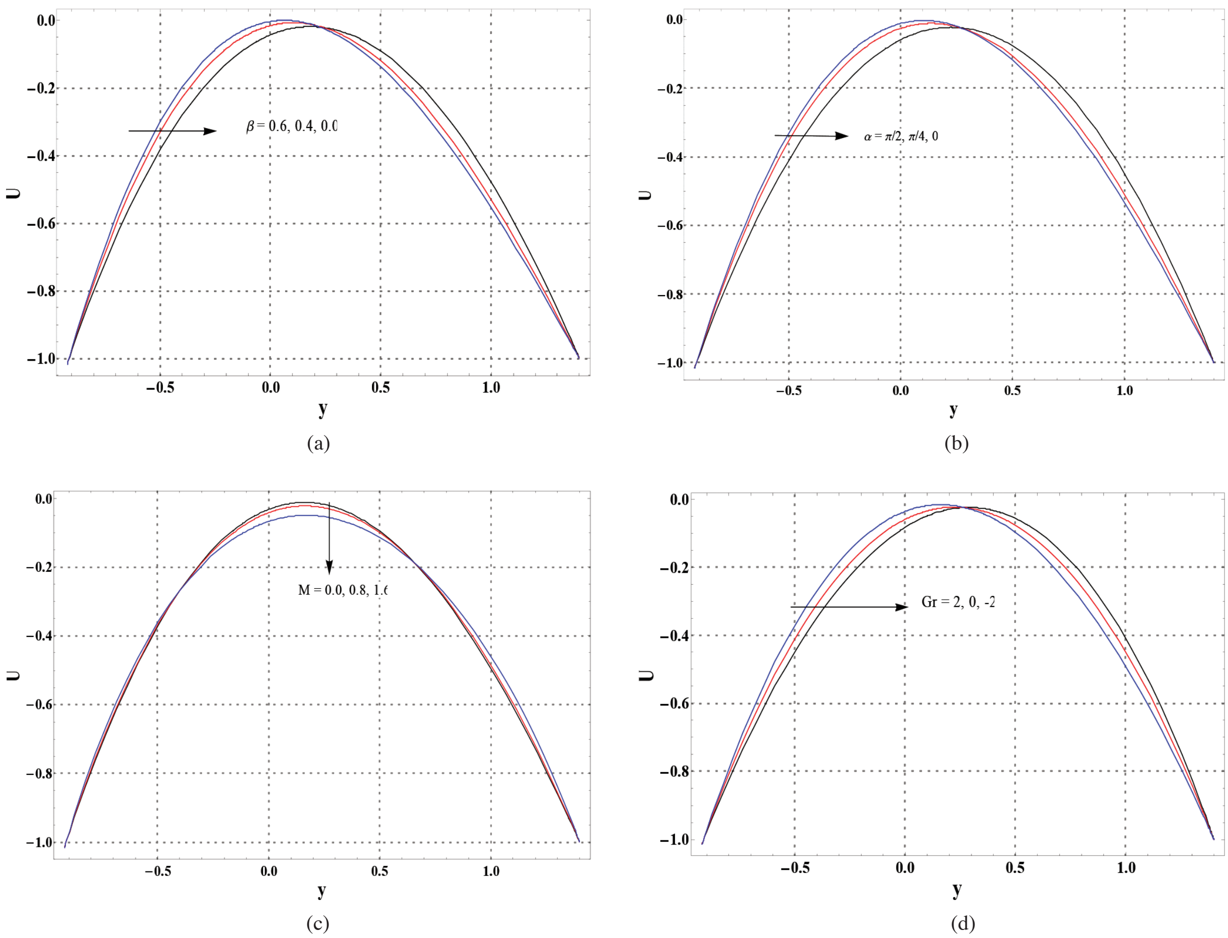

FIGURE 2(a)-2(d). Axial velocity for different parameters

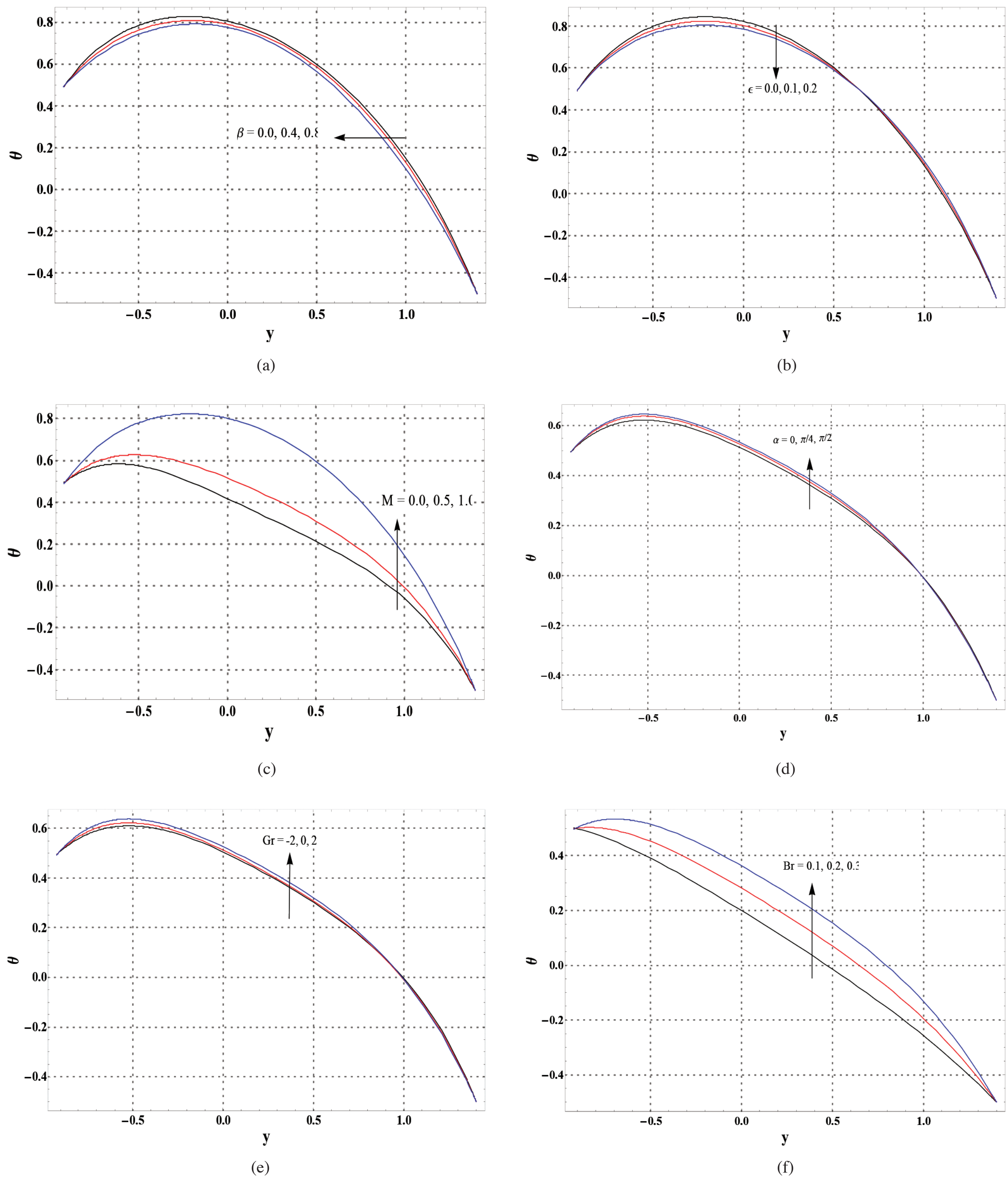

FIGURE 3(a)-3(f). Dimensionless temperature for various parameters

Numerical values of the heat transfer coefficient at the upper wall $(Z = \theta_y (h_1)_x)$ for variation in different parameters is presented in Table 2. Comparison between the results of present analysis and the study proposed by Srinivas and Kothandapani (2008) is provided. For the sake of comparison present study is reduced to a special case when $\beta = 0.0$, $\varepsilon = 0.0$, Gr = 0 and Joule heating is absent. Table 2 shows good agreement between the two results.

RESULTS

The effects of variable viscosity and thermal conductivity in the MHD peristaltic transport of fluid in an inclined

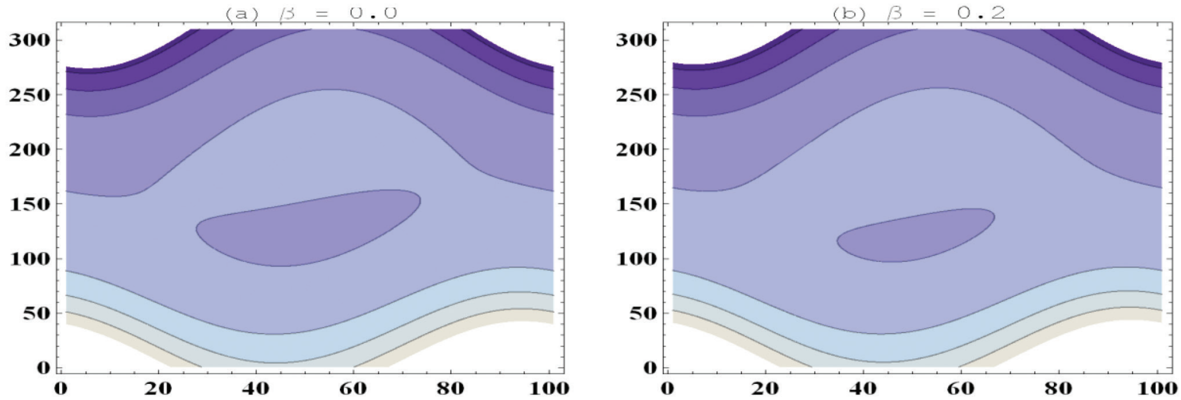

FIGURE 4. Streamlines for viscosity parameter β

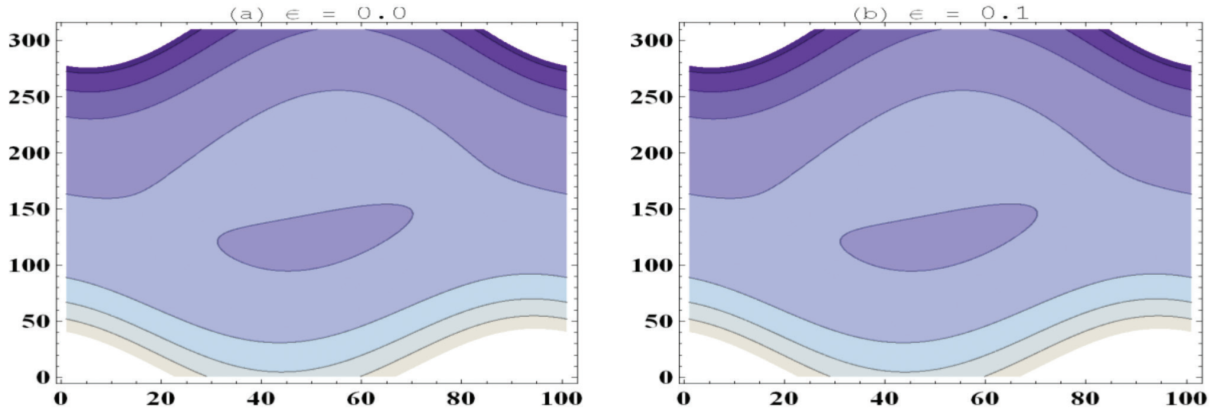

FIGURE 5. Streamlines for thermal conductivity parameter $\in$

TABLE 2. Comparison between numerical values of heat transfer coefficient at the upper wall $(Z = \theta_y (h_1)_x)$ for special case of present study and the work of Srinivas and Kothandapani (2008) when Re = 0, Fr = 2, Gr = 0, Pr = 0.5, a = 0.5, b = 0.6, $\phi = \pi/4$ and $d = 1.5$.

x	M = 2.0		M = 3.0		M = 4.0	
	Srinivas & Kothandapani (2008)	Special case of present study	Srinivas & Kothandapani (2008)	Special case of present study	Srinivas & Kothandapani (2008)	Special case of present study
0.1	1.9440	1.94404	2.1353	2.13528	2.3848	2.38482
0.2	1.8582	1.85821	1.9120	1.91204	1.9920	1.99202
0.3	1.9789	1.97889	1.9932	1.99322	2.0182	2.01823
x	E = 2.0		E = 3.0		E = 4.0	
	Srinivas & Kothandapani (2008)	Special case of present study	Srinivas & Kothandapani (2008)	Special case of present study	Srinivas & Kothandapani (2008)	Special case of present study
0.1	1.5013	1.50131	1.9440	1.94404	2.3868	2.38677
0.2	1.6569	1.6569	1.8582	1.85821	2.0595	2.05952
0.3	1.8692	1.86924	1.9789	1.97889	2.0885	2.08855
x	F = -2.0		F = -1.5		F = -1	
	Srinivas & Kothandapani (2008)	Special case of present study	Srinivas & Kothandapani (2008)	Special case of present study	Srinivas & Kothandapani (2008)	Special case of present study
0.1	1.9440	1.94404	3.6075	3.60752	5.9373	5.93726
0.2	1.8582	1.85821	4.4727	4.47274	9.1554	9.15537
0.3	1.9789	1.97889	2.5390	2.53895	7.6982	7.69817

asymmetric channel are analyzed. Mixed convection and Joule heating effects are present. The results are as follow: Variable viscosity and thermal conductivity tend to increase the pressure gradient; viscosity parameter, Grashoff number and inclination angle have similar effects on the velocity. However, velocity is unperturbed by change in thermal conductivity parameter; constant viscosity fluid has larger bolus whereas there is no effect on bolus size for variable thermal conductivity; variable viscosity tends to reduce the temperature of fluid and viscosity parameter and inclination angle have similar effects on heat transfer rate at the wall.

ACKNOWLEDGMENTS

Fahad Munir Abbasi (second author) acknowledges the financial support from Higher Education Commission (HEC) of Pakistan through PhD Indigenous scheme.

REFERENCES

Abbasi, F.M., Hayat, T. & Ahmad, B. 2014a. Peristaltic flow in an asymmetric channel with convective boundary conditions and Joule heating. *J. Cent. South Univ*. 21: 1411-1416.

Abbasi, F.M., Hayat, T., Alsaedi, A. & Ahmad, B. 2014b. Soret and Dufour effects on peristaltic transport of MHD fluid with variable viscosity. *Appl. Math. Inf. Sci*. 8: 211-219.

Abd Elmaboud, Y. & Mekheimer Kh., S. 2011. Non-linear peristaltic transport of a second-order fluid through a porous medium. *Applied Mathematical Modelling* 35: 2695-2710.

Hayat, T. & Abbasi, F.M. 2011. Variable viscosity effects on the peristaltic motion of a third order fluid. *Int. J. Numer. Methods Fluids* 67: 1500-1515.

Hayat, T., Abbasi, F.M. & Alsaedi, A. 2014a. Soret and Dufour effects on peristaltic flow in an asymmetric channel. *Arab. J. Sci. Eng*. 39: 4341-4349.

Hayat, T., Abbasi, F.M., Ahmad, B. & Alsaedi, A. 2014b. Peristaltic transport of Carreau-Yasuda fluid in a curved channel with slip effects. *PLoS ONE* 9(4): e95070.

Hayat, T., Abbasi, F.M., Alsaedi, A. & Alsaadi, F. 2014c. Hall and Ohmic heating effects on the peristaltic transport of Carreau-Yasuda fluid in an asymmetric channel. *Z. Naturforsch*. 69a: 43-51.

Hayat, T., Abbasi, F.M., Al-Yami, M. & Monaquel, S. 2014d. Slip and Joule heating effects in mixed convection peristaltic transport of nanofluid with Soret and Dufour effects. *J. Mol. Liq*. 194: 93-99.

Hayat, T., Abbasi, F.M. & Hendi, A.A. 2011. Heat transfer analysis for peristaltic mechanism in variable viscosity fluid. *Chin. Phys. Lett*. 28: 044701.

Hina, S., Mustafa, M., Hayat, T. & Alseadi, A. 2013. Peristaltic flow of pseudoplastic fluid in a curved channel with wall properties. *Journal of Applied Mechanics* 80: 024501-1-7.

Latham, T.W. 1966. Fluid motion in a peristaltic pump. MS. Thesis. Massachusetts Institute of Technology, Cambridge, M.A. (unpublished).

Mekheimer Kh, S., Salem, A.M. & Zaher, A.Z. 2014. Peristaltically induced MHD slip flow in a porous medium due to a surface acoustic wavy wall. *Journal of the Egyptian Mathematical Society* 22: 143-151.

Mekheimer Kh., S., Abd Elmaboud, Y. & Abdellateef, A.I. 2013a. Particulate suspension flow induced by sinusoidal peristaltic waves through eccentric cylinders: Thread annular. *Int. J. Biomath*. 06: 1350026.

Mekheimer Kh., S., Komy, S.R. & Abdelsalamd, S.I. 2013b. Simultaneous effects of magnetic field and space porosity on compressible Maxwell fluid transport induced by a surface acoustic wave in a microchannel. *Chin. Phys. B* 22: 124702.

Mekheimer Kh., S., Husseny, S.Z.A. & Abd Elmaboud, Y. 2010. Effects of heat transfer and space porosity on peristaltic flow in a vertical asymmetric channel. *Numer. Meth. Partial Diff. Eqn*. 26: 747-770.

Pandey, S.K. & Tripathi, D. 2012. Unsteady peristaltic transport of Maxwell fluid through finite length tube: Application to oesophageal swallowing. *Appl. Math. Mech*. 33: 15-24.

Shapiro, A.H., Jaffrin, M.Y. & Wienberg, S.L. 1969. Peristaltic pumping with long wavelengths at low Reynolds number. *J. Fluid Mech*. 37: 799-825.

Srinivas, S., Gayathri, R. & Kothandapani, M. 2011. Mixed convective heat and mass transfer in an asymmetric channel with peristalsis. *Communications in Nonlinear Science and Numerical Simulation* 16: 1845-1862.

Srinivas, S. & Kothandapani, M. 2008. Peristaltic transport in an asymmetric channel with heat transfer - A note. *Int. Commun. Heat Mass Transf*. 35: 514-522.

Tripathi, D., Ali, N., Hayat, T., Chuabe, M.K. & Hendi, A.A. 2011. Peristaltic flow of MHD Jeffrey fluid through finite length cylindrical tube. *Appl. Math. Mech*. 32: 1231-1244.

T. Hayat & F.M. Abbasi*
Department of Mathematics
Quaid-I-Azam University 45320
Islamabad 44000
Pakistan

T. Hayat, B. Ahmad & A. Alsaedi
Nonlinear Analysis and Applied Mathematics (NAAM)
Research Group
Faculty of Science, King Abdulaziz University
Jeddah 21589
Saudi Arabia

*Corresponding author; email: abbasisarkar@gmail.com

Received: 13 January 2014
Accepted: 3 March 2014